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REPORT ON
TEST METHODS FOR THE

SINGLE DEGREE OF FREEDOM INTEGRATING RATE GYRO
PHASE I: GYRO CHARACTERISTICS

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INTRODUCTION

Due to the rapid advancement made in the development of inertial navigation components in recent years, new techniques have been devised to determine the performance of gyros. The requirement of greater precision in measurement of gyro characteristics has necessitated that new test equipment be designed, and that sources of error heretofore negligible be considered. As is characteristic of periods of rapid technological progress in a given field, methods of testing for and stating characteristics of gyros vary widely from source to source. There being no standards established relating to the testing for and stating of such characteristics, comparison of different gyros is extremely difficult.

Project TED ADC AE-9205.1 was established for the evaluation of inertial navigation components. In order to accomplish this project an interim facility for inertial testing was established at the Naval Air Development Center (NADEVCEN). Various inertial component test procedures and test equipments are being investigated at this Center.

This report covers single degree of freedom integrating rate gyro test methods, dealing only with the determination of the gyro characteristics other than the gyro drift characteristics. Test methods for the determination of drift characteristics will be the subject of subsequent reports.

SUMMARY OF RESULTS

In the investigation of test methods, various manufacturers and institutions engaged in inertial component development were visited, and test methods were observed and discussed. Technical literature concerned with gyro testing was also reviewed. Once the interim inertial test facility was established, the various methods were scrutinized in more detail. In conjunction with the evaluation of inertial quality single degree of freedom integrating rate gyros, various methods of test were used. In some cases, new test methods were devised.

As a result of this background and study, certain methods were determined to be preferable over others. The criteria for selection are given in Section V of this report. Only methods considered satisfactory are discussed in this report. Details are given under "TEST METHODS." All the methods described are designed for testing the gyro after complete assembly, without disturbing the seal.

CONCLUSIONS

It is concluded that due to the various test methods in existence, and often due to the methods of analysis of test results, comparison of characteristics of gyros is difficult unless methods of test are specified. The methods contained in this report are applicable to single degree of freedom integrating rate gyros where testing is desired after completion of the finished unit.

RECOMMENDATIONS

It is recommended that consideration be given to the acceptance of the methods described in this report as an approach to standardization of methods for determining the performance characteristics of single degree of freedom integrating rate gyros.

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LIST OF SYMBOLS

a_0 Coefficient of the zero degree term of a polynomial

a_1 Coefficient of the first degree term of a polynomial

a_2 Coefficient of the second degree term of a polynomial

C Damping coefficient $\frac{\text{dyne-cm}}{\text{rad/sec}}$

C' Contribution to the damping coefficient from dependence on the square of gimbal angular velocity $\frac{\text{dyne-cm}}{\text{rad}^2/\text{sec}^2}$

d_{Di} i^{th} deviation from the mean of a function due to random drift; a non-stationary, ergodic distribution

d_{Li} i^{th} deviation from the mean of a function due to non-linearity, a stationary, ergodic distribution

d_{Ti} the sum of d_{Di} and d_{Li}

E_o Signal generator output voltage (volts)

f_n Undamped natural frequency (cycles/sec)

F The sum of all constant error torques (dyne-cm)

G_{Tm} Mechanical transfer function (dimensionless)

G_{TS} Signal generator transfer function (volts/rad)

G_{TT} Torquer transfer function $\left(\frac{{}^\circ/\text{HR}}{\text{ma}}\right)$

H Angular momentum $\frac{\text{dyne-cm}}{\text{rad/sec}}$

IA Input Axis

I_t Torquer current (ma)

J Gimbal inertia $\frac{\text{dyne-cm}}{\text{rad/sec}^2}$

LIST OF SYMBOLS
(Cont'd.)

k_s Signal generator scale factor $\frac{\text{volts}}{\text{rad}}$

k_t Torquer scale factor $\frac{\text{dyne-cm}}{\text{ma}}$

K Spring constant $\frac{\text{dyne-cm}}{\text{rad}}$

K' Pseudo spring constant $= \frac{K}{k_s}$ $\frac{\text{dyne-cm}}{\text{volt}}$

OA Output axis

R Static restraints (dyne-cm) or (degrees/hr)

R_{so} Static restraints with the gyro wheel off (dyne-cm)

SA Spin axis

t Time (sec)

T Torque (dyne-cm)

T_{sa} Torque about the spin axis (dyne-cm)

T_{ia} Torque about the input axis (dyne-cm)

T_o Torque about the output axis required to hold a null when $\theta_o = 0$

T_p Torque about the output axis (dyne-cm)

T_t Total torque

[illegible] Mass unbalance along the spin axis (dyne-cm) or (degrees/hr)

[illegible] Mass unbalance along the input axis (dyne-cm) or (degrees/hr)

[illegible] Input axis alignment test angle (rad)

[illegible] The alignment of a reference axis with respect to the horizontal measured in a vertical plane containing the earth's polar axis (rad)

LIST OF SYMBOLS
(Cont'd.)

δ_m	Input axis misalignment with respect to the spin axis (rad)
[illegible]	Input axis misalignment with respect to the output axis (rad)
[illegible]	Misalignment of gyro mount which appears as IA misalignment towards the spin axis
θ	Azimuth angle
[illegible]	Azimuth angle measured so that $\theta' = \frac{\pi}{2}$ where the input axis is horizontal senses zero angular input without compensating for static restraints
[illegible]	Input angle about the input axis
[illegible]	Gimbal output angle as measured by the signal generator
[illegible]	Local latitude angle
[illegible]	Standard deviation of random drift
[illegible]	Standard deviation of (rms) non-linearity
[illegible]	Total standard deviation
[illegible]	Characteristic time (sec)
[illegible]	Applied angular velocity about the input axis
[illegible]	Angular velocity about the input axis (rad/sec)
[illegible]	Applied angular velocity about the spin axis
[illegible]	Gimbal natural angular velocity (rad/sec)
[illegible]	Gimbal angular velocity (rad/sec)
[illegible]	Earth's rotational rate (rad/sec)

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SCOPE

This report is concerned with the testing of single degree of freedom integrating rate gyroscopes. A certain technical familiarity with servomechanisms and gyroscopes is assumed. The scope of this report is limited to tests which determine mechanical and electromechanical characteristics of the gyroscope. In particular, gyro drift tests are not discussed. This subject will be covered in a succeeding report.

The set of tests described is designed to be performed on a completed, sealed gyro. Insofar as possible, the characteristics of the gyro are determined without reference to values quoted by the manufacturer. The only exception in this respect is the angular momentum of the wheel. This value can be measured very accurately during assembly of the gyro and is the value least likely to change.

The more important performance characteristics, such as the damping coefficient, torque generator characteristics, and signal generator characteristics are determined directly. Values required to a lesser degree of accuracy are determined by calculation from relationships derived from the gyroscopic equation of motion for a single degree of freedom gyro (Appendix A). The assumption that a gyro will behave as a system of no higher order than second is implicit in these equations. The effect of higher order perturbations is generally negligible. Certain deviations from equation (1) of Appendix A which do not involve higher order terms are accounted for principally as error sources such as the non-newtonian behavior of the damping fluid.

In general, the tests measure the desired characteristic as directly as possible. Emphasis has been placed on simplicity of test method and equipment. Where possible, extraneous errors are removed by the choice of test method rather than by computation or compensation.

The choice of method for each test was made after careful comparison and refinement of the various possible procedures. All of the test methods described in this report have been used successfully at this Center.

Although modifications of these test methods may be necessary to keep pace with gyro development, the basic test methods should be usable as testing standards for single degree of freedom integrating rate gyros.

The discussions of test methods have been kept as general as possible, in order to apply to all types of single degree of freedom integrating rate gyros. Certain common features are assumed, i.e. the torquer is a direct current sensitive device and the signal generator produces an output voltage related approximately linearly to gimbal angle. Where the gyro design is in conflict with these assumptions, modifications to the various equations and methods may be easily effected.

TEST METHODS

A. Phase Convention

The phase convention is the interrelationship between the mechanical motions of the gyro and the electrical signals associated with these motions. The phase rotation of the spin motor excitation determines the positive direction of the spin axis and is given by the manufacturer of the gyro. The relationships to be determined are the phasing of the signal generator output signal for a given direction of angular input about the input axis, and the polarity of the torquer signal required to close the gyro loop with such an input. These relationships are necessary to define positive and negative outputs from the signal generator and positive and negative command signals to the torquer.

Technique:

The gyro is mounted on a turntable with the output axis vertical and parallel to the table axis. The spin motor is excited with the phasing as specified by the manufacturer. The input axis as defined by the exterior reference on the gyro case is oriented approximately north so that the earth's rotation produces a positive sense of input about the input axis. The gyro is operated in the closed loop mode to bring the signal generator to the null position, then the loop is opened and the phasing of the signal generator output is noted. This phase is then by definition the "positive" output from the signal generator. The gyro loop is then closed through the torquer and the polarity of the torquer signal is noted. This polarity is then by definition the "positive" torque input since it is the torque required to null a positive input about the input axis.

The phase convention determined above is used in all succeeding tests to determine the sign of the signal generator output voltage (E_0) and of the torquer input current (I_t).

B. Torquer Transfer Function

In platform applications it is often desired to give command signals to the gyro. These command signals will usually be proportional to the input angular velocity, in order to maintain platform orientation with respect to the earth or any other orientation program. Therefore, it is necessary to know the function which describes equivalent input rate for a given command signal to the torquer.

At the present time all torquers apply torque by the interaction of magnetic fields. The control of the current in one field of this torquer controls the command torque. The torquer transfer function is defined as the change in input angular velocity with respect to change in torquer current required to hold the gyro at null. This function is normally a constant.

Either of two basic methods may be used to determine the torquer transfer function. A constant input rate may be applied and the torquer current observed when the gyro is operated in the feedback mode, or a constant current may be applied and the table rate observed when the gyro is operated in the servo mode. In order to obtain a representative amount of data from the first method, a quasi-continuous range of precise frequencies would be required for a rate table since frequency control is usually used for precision rates. For the second method, however, a quasi-continuous range of precision current would be required. Since precision currents are more readily obtainable over such a range than are precision frequencies, the servo mode method is preferred.

The alignment of the input axis with respect to the table axis is critical since any misalignment will introduce an error in the known amount of earth's rate proportional to the sine of the misalignment angle. It is also desirable to maintain the plane of the output and spin axes vertical, in order to eliminate effects due to mass unbalance along the spin axis. Thus the input and table axes will be horizontal, and precisely parallel. In order to eliminate effects of mass unbalance along the input axis, the test is performed over an angle whose center is the orientation with the output axis vertical, since the mass unbalance drift will reverse direction when the output axis moves through that point. In general, it will be convenient for the input axis to be directed north, since the servo table is aligned in this direction for other tests. Thus earth's rate will be effective about the input axis. This effect is eliminated by applying rates in both directions about the mean orientation of the output axis.

There would be an exception to the servo method when testing at low rates, which would require a long period of time for each reading. Servo tables which have accurate readouts each one degree, for instance, would require a period of several minutes to determine rates on the order of ten degrees per hour. For rates of this magnitude and lower, the earth may be used as a precision turntable. Any desired rate up to $\Omega \cos\lambda$ may be obtained by positioning the input axis in the horizontal plane such that the input axis is at an accurately known azimuth angle. The angle is chosen to give the desired rate by the equation:

$$\omega_i = \Omega \cos\lambda \ \cos\theta \qquad (1)$$

The absolute orientation of the input axis may be found from the value of spring restraint (R_s = o) determined in the spring constant test. The output axis is oriented vertical so that mass unbalance torques are eliminated.

The range of torques over which the gyro will be tested will of course depend on the particular gyro and its intended application. If the gyro is intended for aircraft inertial systems, the range of equivalent input rates should include values below and about earth's rate. For ballistic applications, higher rates should also be considered. For low rates the random drift rate of the gyro may not be inconsiderable and will often be of the order of magnitude of non-linearity of the torquer or higher. The equation for separating the rms non-linearity and random drift is derived in Appendix C. For accuracy of both the torquer transfer function and the rms non-linearity, a minimum of a dozen data points is considered a sufficient number.

Technique:

High Rates: The input axis is oriented parallel to the table axis accurately horizontal and north. The output axis is rotated about the table axis an angle α off the vertical. The gyro is operated in the servo mode, with various values of precision constant current (I_r) applied to the torquer. The gyro is allowed to servo the table from α to $-\alpha$ and then from $-\alpha$ to α once more. The two traversal times are recorded. The equivalent input rate for each value of current is then

$$\omega_i = \alpha\left(\frac{t_2 \pm t_1}{t_2 t_1}\right) \qquad (2)$$

The positive sign is chosen if the table moves in the opposite direction the second time. The data is then treated by the least squares method with I_t as the independent variable. The slope thus obtained is the torquer transfer function, G_t. The intercept is an indication of the statistical reliability of G_t; the intercept value should be very nearly the value of the static restraint.

The standard deviation (σ_T) from the calculated slope is computed. The rms non-linearity (σ_L) is calculated from

$$\sigma_L^2 = \sigma_T^2 - \sigma_D^2 \tag{3}$$

where σ_D is the standard deviation of the random drift.

Low Rates: The output axis is positioned vertically and the input axis oriented in various directions in the azimuth plane. The gyro is operated in the feedback mode; the torquer current is noted. The input rate is determined by equation (1). The data is treated in the same manner as that for higher rates described above, with the same significance attached to the results.

C. Signal Generator Transfer Function

The method of detecting precession of the gyro in an inertial system application is usually by the detection of voltages generated by a variable reluctance device such as an E type transformer. The signal generator transfer function is defined as the rate of change of output voltage with respect to input angle. This characteristic of the gyro is useful in determining the amount of gain required in servo loop applications. The signal generator transfer function is defined so as to include the effects of the spring restraint and the change in input axis orientation with rotation about the output axis. Therefore, the gyro is considered as a "black box." The effects, however, must be accounted for in determining other gyro characteristics (see Signal Generator Scale Factor), but need not be considered here.

The method of determining the signal generator transfer function is simply to apply angular rates about the input axis and observe the rate of change of signal generator voltage with the gyro operated in the open loop mode.

The gyro could be mounted with the input axis parallel to the table axis with rotation of the table being used to introduce input rates. Under such conditions mass unbalance effects would have to be considered. To eliminate the mass unbalance effects, the output axis may be maintained vertical. The earth itself may be used as the precision turntable. If the output axis is parallel to the table axis and vertical, the turntable may be used to position the input axis such that it is sensitive to any desired portion of the horizontal component of earth's rate. If the gyro is then allowed to drift off null in response to the chosen input rate, the rate of change of signal generator output voltage may be observed and the signal generator transfer function thus determined.

It should be noted that in the open loop mode, as the gimbal rotates about the output axis, the true input axis changes in orientation. Due to this change in orientation, the input axis will sense a varying amount of earth's rate. Usually, this change in orientation will be small and will contribute a slight non-linearity to the signal generator transfer function. This non-linearity is least pronounced near null, and since this is the region of operation in most applications, it is convenient to quote the slope of the curve in this region. For example, in a gyro where $G_{Iw} = 1$, and the gyro input axis is 85° from north, the non-linearity due to change in true input axis orientation will contribute an error of about 0.1% to the null value of G_s if the test time is five minutes.

The input rate chosen will depend principally on the required resolution of time measurement. The rate should be low enough that the accuracy of time interval between voltage readings is sufficient for an accurate determination of the increment of input angle.

b. [illegible]

The gyro is mounted with the output axis vertical and parallel to the table axis. The input axis is positioned to an accurately known [illegible] east or west null such as to give the desired input rate. The gyro is nulled through a feedback loop. The feedback loop is then opened. At preselected intervals of time, the signal generator voltage is recorded.

For each reading the total time is used to calculate the total input angle by the equation:

$$A_i = t(\text{[illegible]})\qquad (4)$$

The least squares slope is then calculated (see Appendix B) with the observed signal generator voltage as the dependent variable. This slope is G_n. Care should be exercised that the slope is representative of the [illegible] portion of the curve which is representative of the normal operation of the unit.

D. Input Axis Alignment

Since inertial systems use orthogonal coordinates, it is necessary to know the precise direction of the input axis of the platform control gyros. The input axes must form an orthogonal set in order to eliminate cross-coupling of input rates. For mounting purposes, the input axis must be defined with respect to a reference on the exterior of the gyro. The reference may be a plane to which the input axis is normal, or a notch which defines the input axis in a given plane. The accuracy to which such a reference defines the true input axis of the gyro is the input axis alignment accuracy.

The determination of misalignment of the input axis with respect to both the output and spin axes is desirable, since this will give the magnitude of the cross-coupling effect directly. The Cartesian angles which are determined below are small, and may therefore be assumed to be Eulerian angles also.

Since the single degree of freedom gyro is sensitive only to components of angular rotation about its input axis, rotation about the output and spin axes should not produce an angular displacement of the gimbal assembly with respect to the case. If the true input axis is misaligned with respect to the output axis an amount δ_p, or to the spin axis an amount δ_m, components of the rotation applied about these axes will be sensed by the true input axis in a magnitude $\omega_i = \omega_m \sin \delta_m$ or $\omega_i = \omega_p \sin \delta_p$. which for small angles is $\omega_i = \omega_m \delta_m$ or $\omega_i = \omega_p \delta_p$. To hold errors to a minimum it is not necessary that precise alignment of the rotation vector with respect to the output or spin axis be achieved, only that the rotation be precisely orthogonal to the reference input axis. A misalignment of two degrees with respect to the insensitive axes will contribute an error of only 0.06% to the misalignment angle, as long as the rotational input is orthogonal to the reference input axis.

In order to obtain a test procedure which eliminates as many extraneous effects as possible, consider the major error sources. Consider a general orientation obtained in the following manner (See Figure 1). The spin axis is initially horizontal north (SA_1) with the output axis vertical(OA_1). If the gyro is rotated an angle β about the reference input axis (IA_1), then through an angle α about the new position of spin axis (SA_2), the general orientation (SA_3; IA_3) will be shown in Figure 1. Note that a test angular rate ω_m about the spin axis (SA_2) will change the angle α . For a test rate ω_m about the spin axis (SA_2), the total effective input rate will be:

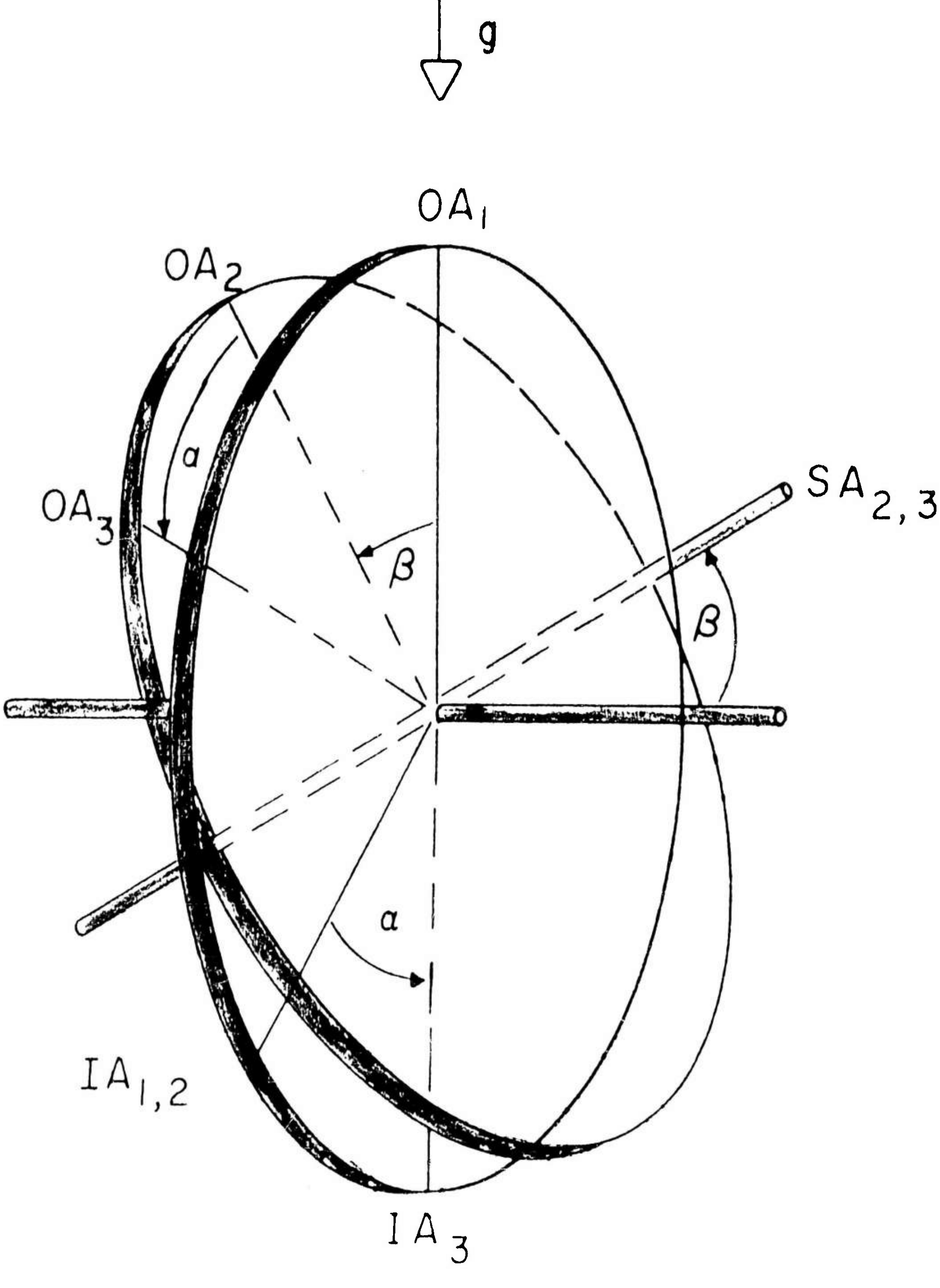

FIGURE 1.
[illegible] OF THE [illegible] AXIS ALIGNMENT TEST

$$\omega_i = \omega_m(\delta_m + \epsilon_m) + (R + U_n \sin\beta + U_m \cos\beta \sin\alpha) + \Omega \sin\alpha \sin(\lambda - \beta) \quad (5)$$

where ϵ_m is the misalignment of the spin axis with respect to the test table axis, in the direction of the reference input axis (see Figure 2), U_m and U_n are mass unbalance terms along the spin and input axes, respectively, and Ω is the earth's rate.

Note that there is no orientation for which the unbalance terms disappear. The functions in table angle α about the spin axis are odd functions, however, therefore they may be averaged to zero by driving the table at a constant rate ω_m from $+\alpha$ to $-\alpha$, giving an average effective rate of:

$$\omega_i = \omega_m(\delta_m + \epsilon_m) + (R + U_n \sin\beta) \quad (6)$$

In practice, the angle β is chosen as nearly as possible equal to zero. If the table is then rotated in the other direction, from $-\alpha$ to $+\alpha$

$$\omega_i' = -\omega_m(\delta_m + \epsilon_m) + (R + U_n \sin\beta) \quad (7)$$

then the restraint and mass unbalance terms may be extracted, giving

$$\delta_m + \epsilon_m = \frac{\omega_i - \omega_i'}{2\omega_m} \quad (8)$$

If the gyro is then rotated 180^0 about the spin axis in the mount, the contribution from mount misalignment will be reversed, giving

$$\delta_m - \epsilon_m = \frac{\omega_i - \omega_i'}{2\omega_m} \quad (9)$$

and this error source may be eliminated.

A similar analysis for rotation about the output axis will yield equations similar to (8) and (9), with the desired mean orientation being output axis vertical and input axis west.

These tests may be performed in the feedback mode of gyro operation with the feedback current monitored. The input is applied as a constant

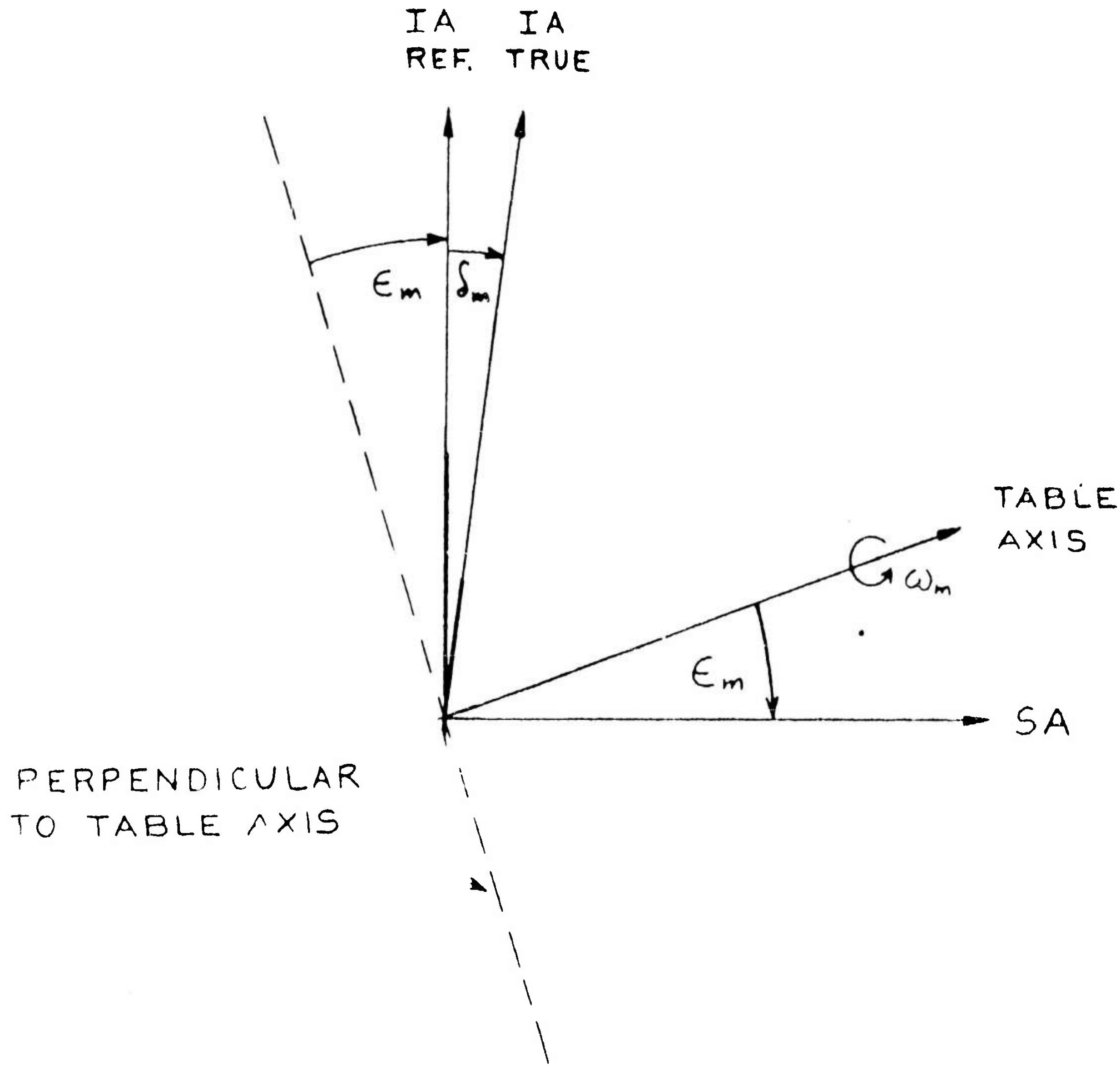

FIGURE 2.
SHOWING RELATION OF INPUT AXIS MISALIGNMENT AND MOUNT MISALIGNMENT

angular rate which must be of a fairly high magnitude. For instance, to determine ten seconds of arc misalignment to two significant figures, assuming the torquer current is accurately read to the equivalent of 0.001°/hr., the minimum table rate necessary is 200°/hr. Rates used in tests of this type are usually much higher in order that the earth's rate contribution may be averaged by the relatively slow response of the readout device. Also the time over which the output is averaged should be small so that random drift is not a consideration, yet an integral number of cycles of the table should be included. These considerations also necessitate a higher rate for the table. This method may prove damaging to the gyro. High rates about the output axis cause induced torques on the output bearings which are not accompanied by an output from the signal generator. Large forces may thus occur at high angular rates which may seriously damage the output axis bearings.

Instead of considering the input and output torques, or the equivalent rates in the feedback mode of operation, an open loop method may be used. Rather than rates, their time integrals, the total angles, are now considered. For this method it is still necessary that the time be small. To eliminate the need for an integral number of cycles, the tests are performed in the orientation where effective earth's rate and effective mass unbalance are small and reverse sign. The mean orientation required for angular rate about the output axis is output axis vertical and input axis east. The mean orientation required for angular rate about the spin axis is spin axis horizontal north and output axis vertical. If the table is moved through a small angle centered at this mean orientation in the open loop mode, the effects of mass unbalance and earth's rate will be nullified. The table would move at a constant rate since it is the time integrals of these quantities which must cancel. For a gyro with a mechanical transfer function of ten, an increment of ten degrees ($\pm 5^{\circ}$) is sufficient to determine the misalignment within one second. It is necessary to apply a fairly high rate in order to reduce the time of travel, but this rate need not exceed 200°/hr. for the instance above so that the total time is three minutes. In general, an accuracy of one second in measurement of the misalignment is not necessary. Further, the wide variation in transfer functions will affect the choice of angles and rates to be used in the test. Although higher rates will not be damaging when applied about the spin axis, rates about the input axis should be below 400°/hr.

The open loop method is preferred principally because of the much lower input rates which may be used. The technique described below has been used to determine misalignment within 3 seconds. The input was applied manually through the fine adjustment of a dividing head, obviating the necessity of using a motor driven table.

Technique:

The gyro mount is aligned with the input axis normal to the table axis. The spin axis is parallel to the table axis and horizontal north. The table is positioned such that the input axis is west, then moved through the angle α (see Figure 1).

The torquer loop, which was closed to maintain the gyro at null, is opened. The table is then turned at a uniform rate through the angle 2α, positive about SA, the input axis passing through west. The output of the signal generator (E_o1) is read immediately. The torquer loop is closed once more to bring the gyro to null. Open the torquer loop. Now reverse the direction of table rotation so as to move from $-\alpha$ to α, and record E_o2. Then rotate the gyro in the mount 180° about the spin axis and repeat the entire procedure, recording E_o3 and E_o4.

Care must be taken in each case to record the phase of the output voltage in accordance with the phase convention determined in Section A "Phase Convention." The misalignment of the input axis with respect to the spin axis is then given by:

$$[\text{illegible}] = \frac{E_{o1} - E_{o2} + E_{o3} - E_{o4}}{8 G_o \alpha} \qquad (10)$$

Now align the table axis vertical with the output axis of the gyro parallel to the table axis. The table is positioned such that the input axis is west, then moved through the angle α. Repeat the above procedure to determine δ_{α}.

E. Spring Constant

The spring constant is the rate at which the torque about the output axis changes with gimbal angle for a constant (zero) torque about the true input axis. This torque arises principally from the flex leads which are used to convey electrical signals from the outer case of the gyro to the floated gimbal. These flex leads behave as tiny springs and give rise to spring-like torques (spring restraint) which are proportional to gimbal angle. The spring constant is also of value in determining the servo dynamics of a loop in which the gyro is used.

The unstrained position of the flex leads does not in general coincide with the gimbal null as defined by the signal generator output. If we define the gimbal output angle as measured by the signal generator as θ_O, and define the gimbal output angle where the flex lead restraints are zero as θ_{On}, then the equation of motion of the gimbal is -

$$J\ddot{\theta}_o + C\dot{\theta}_o + K\theta_o - K\theta_{oN} = H\omega_i \qquad (11)$$

where the last term on the left is a constant. This is what is commonly called the static restraint $R_{s=0}$ for the wheel off. If we then consider the gyro equation of motion with the gimbal motionless, the internally generated torque is -

$$T = K\theta_o + R_{s=0} \qquad (12)$$

If we then measure the torque necessary to hold the gimbal at given angles by using the feedback loop, the static and spring restraints may both be determined.

In general, the gimbal angle will not be known since the signal generator scale factor cannot be determined without some knowledge of the spring restraint (see Section F "Signal Generator Scale Factor"). This is not a circular argument, however. The units of the torque term $K\theta_O$ must be in dyne-cm. Since in the actual test for spring restraint the gimbal angle may be uniquely determined by the signal generator output voltage, we may define a pseudo-spring constant K^1 as the torque per volt of signal generator output. Thus the above equation becomes -

$$T = K'E_o + R_{s=0} \qquad (13)$$

Once K^1 has been determined, the torques arising from the spring restraint in the test for signal generator scale factor (Section F) can be computed. When the signal generator scale factor has been determined, K may be computed from -

$$K = K'k_s \qquad (14)$$

These manipulations do not affect $R(s = o)$, since this value is calculated as a torque and requires no conversion.

Technique:

The gyro is mounted with the output axis vertical and parallel to the turntable axis. The gyro wheel remains unenergized. Using a feedback loop, the torque required to null the signal generator output voltage is recorded. The feedback loop is then operated so that the gyro is servoed to various signal generator output voltages selected to be representative of the range of gimbal angle, with proper regard to signal generator phasing.

The pseudo-spring constant K^1 is then the slope of the lease squares line with ε_o as the independent variable. The torque intercept $R(s = o)$ is the static restraint and should be very closely the value of the torque measured at signal generator null, which is a check on data reliability.

Once the signal generator scale factor is determined, the true spring constant is calculated from -

$$K = K' k_s \tag{14}$$

F. Signal Generator Scale Factor

The signal generator scale factor is defined as the rate of change of output signal, usually a voltage from the signal generator with respect to change in gimbal output angle. Although the signal transfer function is of more use in platform computations, the signal generator scale factor is required for computation in other tests for gyro characteristics and is of inherent value in itself in determining compliance with specifications.

There are three difficulties inherent in measuring this value. The first is that there is no exterior mechanical reference on most gyros which will give any quantitative indication of gimbal movement. The second is that the spring restraint, being a linear function of the gimbal output angle, introduces a bias function when trying to determine the signal generator scale factor. Third, any change in gimbal angle changes the orientation of the spin vector and thus the true input axis.

The following development leads to a technique which has been used successfully with gyros having low random drift rates (below 0.01°/hr.). Attempts to use this technique with gyros of higher drift rate have been aborted since the random drift tends to mask the small quantities involved.

The steady state equation of motion for a gyro in a stable orientation may be written:

$$T = H\omega_i + K\theta_o + F \qquad (15)$$

where F is the sum of all constant torques such as restraints and those due to mass unbalances. Assume an orientation with the output axis vertical and the input axis an angle θ from north. Then

$$T = H\Omega\cos\lambda\cos\theta + K\theta_o + F \qquad (16)$$

The torque may be measured by maintaining the gimbal position by the feedback loop and measuring the torquer current. If the gimbal is held at null by the feedback torque, the spring restraint term is zero. If a bias is set into the feedback loop so that the gimbal is held off null an angle $d\theta_o$, then the true input axis will be misaligned in the azimuth plane by an angle $d\theta = d\theta_o$.

The change in torque is then

$$dT = -H\Omega\cos\lambda\sin\theta\, d\theta_o + K d\theta_o \qquad (17)$$

The unknown quantities in the last equation are K and $d\theta_o$, and it is assumed that the signal generator voltage corresponding to $d\theta_o$ is determined by the bias in the feedback loop. K is not known explicitly, but the entire torque term $Kd\theta_o$ may be determined from the pseudo-spring constant determined by the test of Section E "Spring Constant." Since the spring restraint torque is $K'E_o$, then

$$dT_K = Kd\theta_o = K'dE_o \qquad (18)$$

The pseudo-spring constant K^1 having been determined previously and given as one of the controlled variables of the test, the increment of gimbal angle is then, substituting increments for infinitesimals,

$$\Delta\theta_o = \frac{\Delta T_K - \Delta T}{H\Omega\cos\lambda\cos\theta} \qquad (19)$$

With regard to the choice of azimuth angle θ to be used in the test, it is desirable that $\frac{dT}{T}$ be as large as possible, since the tendency is for the value to be small normally. The condition for θ is then $\theta = 90°$. Thus the input axis should be nearly east.

Technique:

The gyro is mounted on a turntable with the output axis vertical and parallel to the turntable axis. The input axis is oriented east and the gyro is nulled through the feedback loop. The torque required to maintain the signal generator at a null is recorded as T_0. A bias voltage is inserted in series with the signal generator output voltage so that the gimbal position would be offset from the normal null position, by angles selected as representative of the gimbal range. The feedback torque (T_t) is recorded for each such orientation of the gimbal. For each signal generator voltage chosen, the gimbal angle is calculated from -

$$G_o = \frac{K'E_o - T_t + T_o}{[illegible] \sin\theta} \qquad (20)$$

The signal generator scale factor may then be calculated as the slope of the least squares line with the gimbal angle as the independent variable and the signal generator voltage as the dependent variable. The voltage intercept should approximate the gimbal null voltage.

C. Damping Coefficient

The integrating characteristic of a gyro arises from the use of a viscous damping fluid. This fluid is often used for floatation of the gimbal assembly as well. The damping coefficient, C, is the measure of the torque produced by the viscous shear of the fluid as the assembly moves through it. The magnitude of this coefficient determines the mechanical transfer function of the gyro for a given angular momentum. In an inertial system which uses the gyro signal to drive the gyro platform to a null, the damping coefficient is [illegible] importance in its effect on the Schuler tuning of the [illegible].

Since the shear of the damping fluid appears as a torque about the output axis, a torque applied about the output axis is balanced by the torque exerted by the fluid with a constant output axis angular velocity. The damping coefficient can be expressed analytically by -

$$C = \frac{dT}{d\omega_0} \qquad (21)$$

Thus by applying various known torques about the output axis through the torquer and observing the output angular velocity, the damping coefficient may be calculated.

It is important that the torque be accurately known. For this reason the orientation of the gyro in test should be such as to eliminate as many sources of error torque as possible, that is, the output axis should be vertical to eliminate mass unbalance effects. The orientation of the input axis need not be considered since the test may be performed with the wheel off. The presence of wheel spin does not affect the damping in any way, and this eliminates exterior angular rates.

In order to eliminate error torques such as static restraints and spring restraints, the torque is usually applied first in one direction and then the other, and the corresponding output angular velocities are averaged. The gimbal displacement range is usually centered at null to take advantage of a wide gimbal angle for test.

Technique:

The gyro is aligned with the output axis vertical and the gyro wheel remains off. An accurately known torque is applied about the output axis through the torquer. This torque is applied after the gimbal has been offset by an angle greater than θ_0 so that the gimbal will be rotating at constant angular velocity when it passes through the angle θ_0, thus eliminating gimbal inertia effects. The time is recorded which is required by the gimbal in moving through an angle $2\theta_0$, from $+\theta_0$ to $-\theta_0$ as measured by the signal generator. The test is repeated with the same torque moving the gimbal through the angle $-\theta_0$ to $+\theta_0$. The entire procedure is then repeated with other values of torque to obtain at least eight sets of data. The range of torques should be selected so that the operating range of the gyro is adequately represented. For each value of torque, the angular rate is then computed by -

$$\omega_0 = \theta_0 \left(\frac{t_1 + t_2}{t_1 t_2} \right) \qquad (22)$$

where t_1 and t_2 are the traversal times in each direction. The torques (T) and corresponding angular rates (ω_c) are then used to calculate the damping coefficient by the slope of the least squares equation (Appendix D).

$$C = \frac{\sum T\omega_c - n\bar{\omega}_c\bar{T}}{\sum \omega_c^2 - n\bar{\omega}_c^2} \tag{23}$$

where T is the torque produced by the torquer ($T = k_T I_T$).

H. Dynamic Friction

The dynamic friction is a measure of the dissipative torques which do not produce angular motion of the gimbal about the output axis. The data for determining this value is the same as obtained in the test for determining the damping coefficient (Section G "Damping Coefficient"). The intercept of the least squares line on the torque axis as obtained from that data is the dynamic friction torque. Care must be exercised, however, that the intercept so obtained is representative at the gimbal angular velocity of zero. Thus, the data should include low angular velocities and the intercept determined in the linear region near zero angular velocity, so that the dependence of the damping coefficient on higher degree terms of the power series is negligible.

I. [illegible] Behavior of the Damping Fluid

In determining the damping coefficient (Section G) it was assumed that the viscous torque is a linear function of gimbal angular velocity. Such a fluid is known as a Newtonian fluid. In fact, however, the viscous fluid will not so behave. If the shear torque depends on higher degree terms of the gimbal angular velocity, the mechanical transfer function, and hence the servo gain, will not remain constant at high angular rates.

The dependence of the damping torque on gimbal angular velocity may [illegible] be written in terms of a rapidly convergent power series. Due to the rapidity of convergence, a figure of merit chosen is the relative dependence of the damping torque on the square of the gimbal angular velocity. The shear torque may be written as

$$T' = a_0 + a_1\omega_0 + a_2\omega_0^2 \tag{24}$$

where a_0 is the dynamic friction (Section H) and a_1 is the damping coefficient (C)(Section G). Let a_2 be called C'. The corresponding equation for a Newtonian fluid is

$$T = a_0 + a_1\omega_0 \tag{25}$$

Thus the error torque is

$$T' - T = \Delta T = C'\omega_0^2 \tag{26}$$

and

$$\frac{\Delta T}{T} = \frac{C'}{C}\omega_0 \tag{27}$$

ignoring the dynamic friction as a constant. The factor $\frac{C'}{C}$ in Equation (27) is the figure of merit desired and gives the relative error torque as a linear function of the gimbal angular velocity.

The method of calculating a_0, C, and C' for the assumed form of the torque function as in (24) is given in Appendix E.

J. Change in Damping Coefficient with Operating Temperature

The viscosity, and thus the damping coefficient, of the damping fluid in a gyro is usually quite sensitive to temperature. In many cases the gyro case contains a heating element which is designed to maintain the damping fluid in a narrow range of temperatures. Moreover, a stable platform is usually temperature stabilized. Thus the dependence of the damping coefficient on gyro operating temperature is an important design figure in temperature controllers.

Most gyros which have integral heaters use temperature sensitive resistors in the gyro as a fourth element in a bridge control circuit. In order to change the operating temperature, it is necessary to pad the sensing resistor so that the bridge balances at the desired temperature. In order to select the proper padding resistance for a given temperature, it is necessary to know the temperature coefficient of the sensing resistor.

After choosing the desired padding resistors, the test for damping coefficient is repeated at the various temperatures. The change in damping coefficient with operating temperature is thus given by the least squares slope of damping coefficient versus operating temperature.

K. Angular Momentum

The angular momentum of the gyro wheel is the principal measure of gyroscopic action and in combination with the damping coefficient gives the physical relation between input and output angles. The angular momentum is defined as the product of the moment of inertia of the gyro wheel and its angular velocity. For use in the gyroscopic equation of motion, it is convenient that the units of the angular momentum be $\frac{\text{dyne-cm}}{\text{rad/sec}}$.

It is impossible to measure the angular momentum of the wheel accurately once the gyro has been assembled. The value is usually precisely quoted by the supplier and it is preferable to accept that value in all computations, unless the gyro is to be disassembled.

L. Torquer Scale Factor

The torquer scale factor is the ratio of change of torque with respect to change of torquer command signal. Torquer command signal is usually defined in terms of direct current (See Section B). This value is not independently valuable if the torquer transfer function is known. It is related directly to that value and may be calculated from it by the equation:

$$k_T = G_{Tr} H \qquad (28)$$

M. Mechanical Transfer Function

The mechanical transfer function is defined as the ratio of gimbal output angle to angular input about the input axis. The value may be computed from

$$G_m = \frac{H}{C} \qquad (29)$$

This value is the mechanical gain through the gyroscope and is, of course, dimensionless.

N. Characteristic Time

The time of response of the gyro is of prime importance in the design of the servomechanism which is the inertial platform. The measure of the response time of the gyro is the characteristic time, which is defined in a manner similar to the characteristic time of other electro and/or mechanical systems.

The characteristic time is defined as that time required for the gimbal output angle to reach $\frac{1}{e}$ of its steady value after a step input angle. This definition is then consistent with the concept of characteristic time for other systems. As is often the case with mechanical systems, the step input is difficult to obtain in the laboratory for displacements. Since the gyro behaves as a second order system, the ramp input may be used to determine the characteristic time. Thus if a ramp function of input angle is applied after a time large compared to the characteristic time, the function of output angle, and thus output voltage, will be a ramp displaced along the time axis by the value of the characteristic time. Thus, if the linear function of output angle is determined, the intercept on the time axis is the characteristic time.

The ramp function of torque may be easily obtained by applying a constant current to the torquer at time zero. The inductance and resistance of the torquer should be measured and the characteristic time of the torquer computed. This value should be several orders of magnitude smaller than the expected value of the characteristic time of the gyro. This should also apply to any switching devices used in the measurement.

Technique:

The gyro is mounted with the output axis vertical and the input axis is oriented so that there is no input rate to the gyro. The feedback loop is opened and simultaneously a constant current is applied to the torquer. The act of switching the constant current on is used to trigger a single sweep on an oscilloscope. A camera whose shutter is open during the operation photographs the trace.

The amplitude of the trace and corresponding times are recorded from the photograph over the region exceeding three times the value of the expected characteristic time. The intercept of the least squares line of time as a function of amplitude is obtained. Although the least squares method is sometimes unnecessary, it will give greater accuracy, particularly when the signal generator carrier frequency has any degree of modulation. A sample photograph is shown in Figure 3.

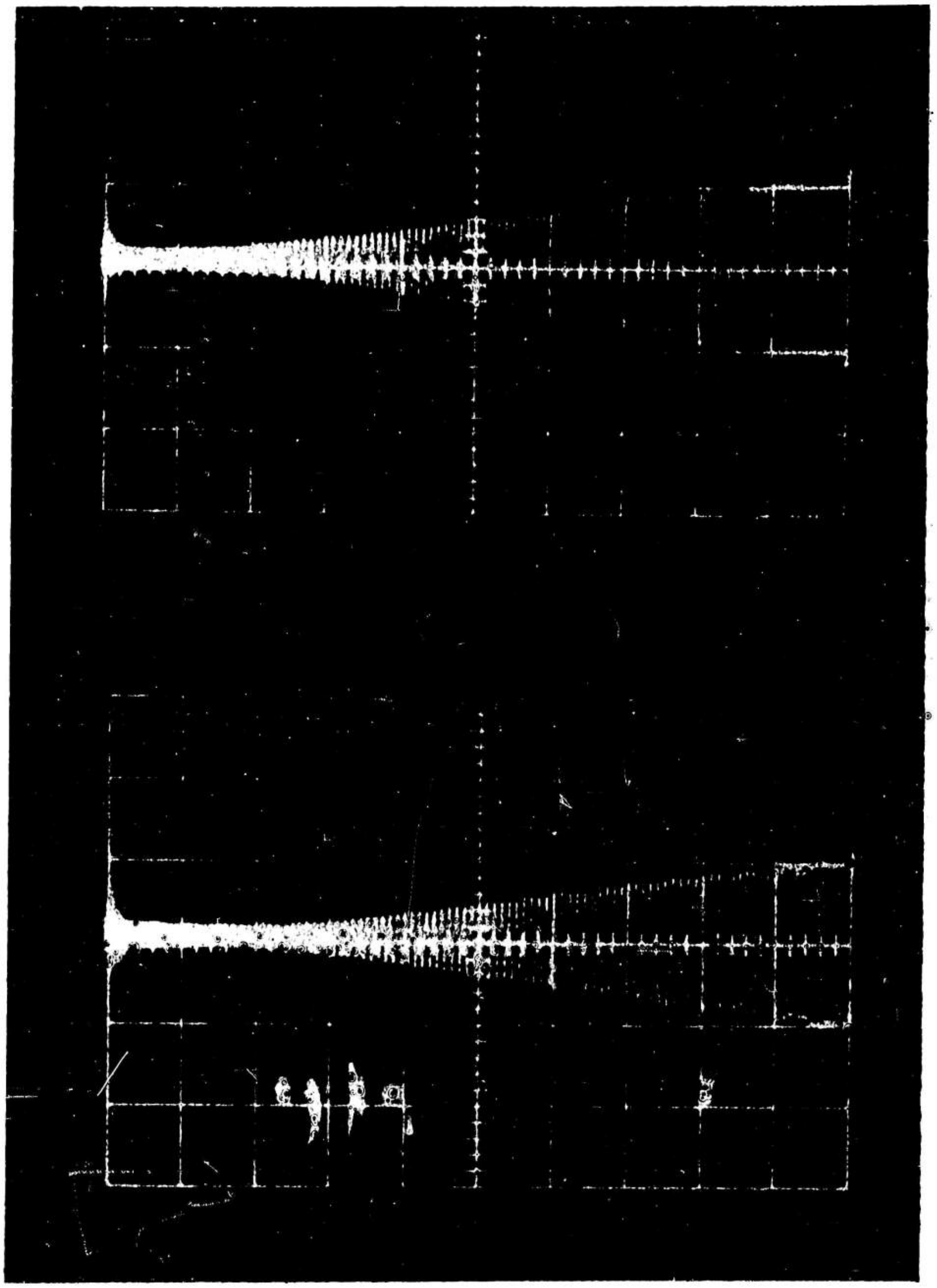

FIGURE 3
TYPICAL PHOTOGRAPHS OBTAINED FROM CHARACTERISTIC TIME TEST

O. Gimbal Inertia

The inertia of the total gimbal assembly is easiest determined before the gyro is assembled. It may be calculated, however, once the damping coefficient and the characteristic time have been determined. From the equations of motion (Appendix A):

$$\tau = \frac{J}{C} \qquad (30)$$

$$J = C\tau \qquad (31)$$

P. Natural Frequency (Undamped)

The undamped natural frequency is the frequency at which the gimbal would oscillate in the absence of damping. It is of value in servo analyses involving the gyro, because in servomechanisms second order systems are usually characterized by the damping ratio and the undamped natural frequency.

Although both the damping ratio and the undamped natural frequency may be determined from complex dynamic tests, it is easier to calculate the undamped natural frequency from equation as derived in Appendix A.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{J}} \qquad (32)$$

Q. Damping Ratio

The damping ratio is defined as the ratio of the damping coefficient to the critical damping. This value is useful for analysis of the gyro in a servo loop. The damping ratio may be calculated from previously obtained data.

$$\zeta = \frac{C}{\sqrt{4JK}} \qquad (33)$$

TEST EQUIPMENT

A discussion of the equipment necessary to perform the tests discussed herein must of course be restricted to generalities. The particular requirements of test and operational equipments will vary widely from gyro to gyro.

For the measurement of electrical signals, various voltmeters, ammeters, and phase meters are required. With the present trend toward minute torquer control currents (G_T), a microammeter is essential. A precisely regulated source of constant direct current and/or voltage is necessary for such tests as that to determine the damping coefficient. The oscilloscope used in the determination of the time constant of the gyro should have a negligible rise time compared to the time constant being measured and should have provisions for external triggering of a single sweep.

The characteristics of the wheel supply and the heater controller, if any, will be specified by the manufacturer of the particular gyro.

A servo amplifier is necessary to provide command signals to the torquer in response to the output of the signal generator, such that output is held near a null. The form of such an amplifier for a gyro which has a d.c. torquer and an a.c. signal generator is usually an amplifier-demodulator combination. The principle requirement of the servo amplifier is that the gyro be held as nearly at null as possible for the given conditions to be imposed. For the spring constant and signal generator scale factor tests the servo amplifier should include provisions for holding the gyro to selected off-null positions. Electronic damping must usually be provided in the servo amplifier, particularly for gyros which have a large value of G_m, and thus a large mechanical gain factor.

The most general test stand used in these tests is a precision turntable which allows adjustment of the gyro about two axes. The tables used are dividing heads such as that shown in Figure 7, having angular readouts about the table axis accurate to values ranging from two to ten arc seconds.

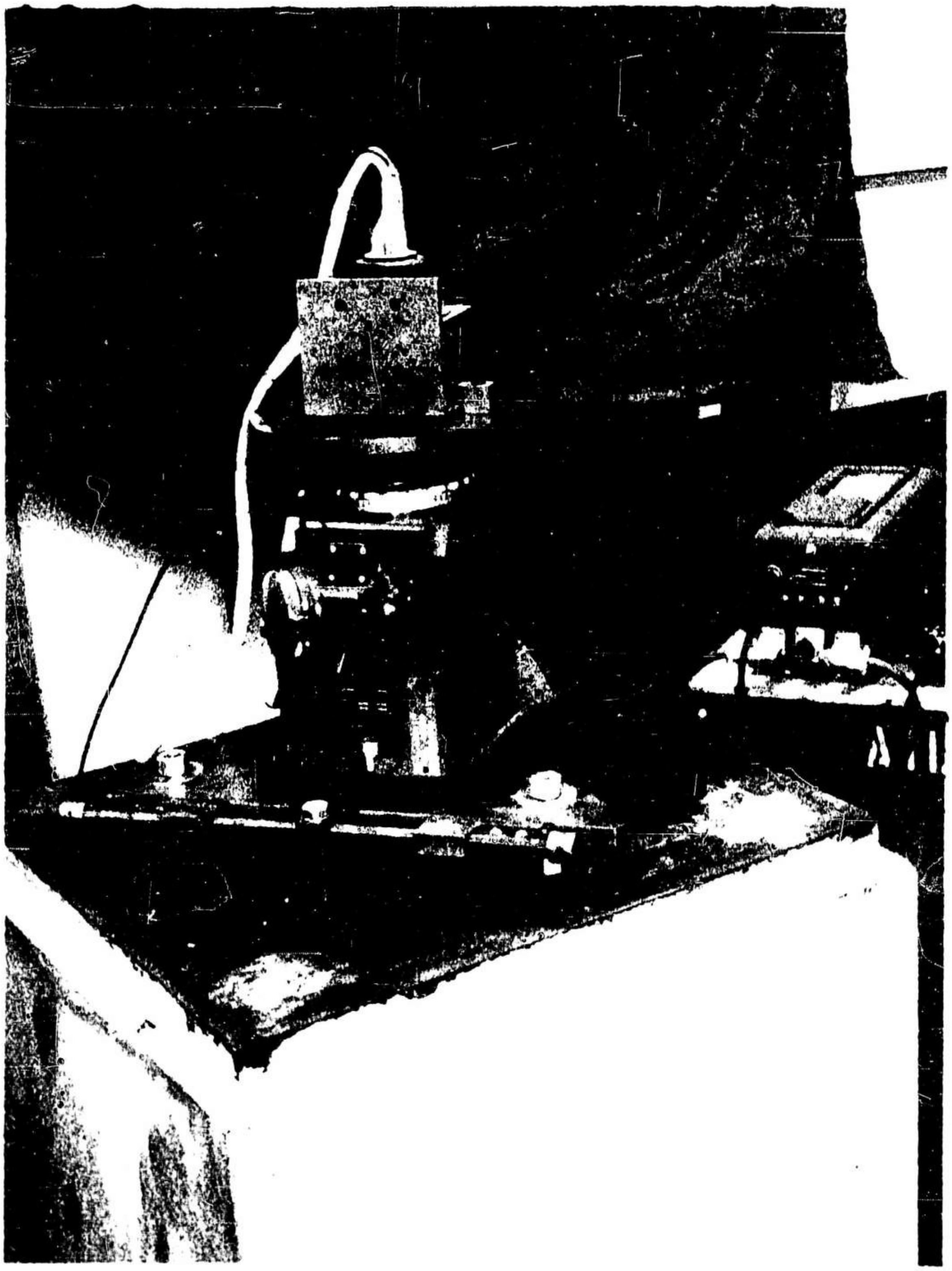

FIGURE 7.
TYPICAL DIVIDING HEAD TEST TABLE

A servo table is used in the torque generator transfer function test. A servo table provides an input angular rate to the gyro in response to a signal from the gyro signal generator in order to maintain that signal near null. The requirements on this table and the associated electronics are similar to those on the servo amplifier used in the torquer mode of operation. The table must also be provided with a precision readout of angular position. Although rate tables are not used in the tests described in this report, they are useful in tests which will be described in future reports.

Although dynamic characteristics of the gyro, such as the damping ratio, the undamped natural frequency, and the characteristic time may be determined using test stands which introduce programmed dynamic mechanical motions, there are many problems with even the simplest of these. For most purposes, the accuracy obtained by calculating the damping ratio and undamped natural frequency from previously obtained values is sufficient, in that these values are used for the design of the basic platform servo loop and are not generally required to be precise values.

GROUPING OF TESTS AND ORDER OF PERFORMANCE

Although the order of performance of the described tests is flexible, certain values obtained will be useful in other tests which are not described in this report. Furthermore, certain tests, due to common features such as orientation on the test table, may be conveniently grouped together. The preferred grouping and order of performance is given below. Justification for the selection then follows.

I. Phase convention
II-A. Torquer transfer function
II-B. Signal generator transfer function
III. Input axis alignment
IV-A. Spring constant
IV-B. Signal generator scale factor
V-A. Damping coefficient
V-B. Change in damping coefficient with operating temperature
VI. Characteristic time

All other values are calculated from the data on results of these tests.

The phase convention test is performed first because therein are determined the positive and negative sense of the signal generator and torque generator signals. The low range torque generator and signal generator transfer functions are grouped together because these values will be necessary for other tests to be performed which are not included in this report. In particular, certain drift information would be desired early in the life of the gyro and would follow the tests in Group II. Information concerning the alignment of the input axis is not necessary for these tests because they are referenced to zero inputs to the gyro arbitrarily. The input axis alignment test is performed next, however, so that proper alignment procedures may be effected should the need arise.

The spring constant test and signal generator scale factor test are grouped together because their values are related to each other. Moreover, both tests require the same gyro orientation and the use of special electronic equipment to maintain the gyro loop at some off-null signal generator voltage.

The damping coefficient tests are logically grouped together and are followed by the characteristic time test.

Note that all tests after the input axis alignment test, except for the signal generator scale factor test, may be performed with the gyro wheel off. This effectively lengthens the life of the gyro, thus allowing more time to be spent on drift tests or allowing the gyro to be used in a stable platform for a reasonable time after undergoing tests.

APPENDIX A

Single Degree Of Freedom Gyroscopic Equation Of Motion

The torque produced by a single degree of freedom gyro is given by the product of the angular momentum and the input angular velocity, $H\omega_i$. Ideally, in the absence of error torques, this torque must overcome the inertia reaction torque of the gimbal, $J\ddot{\theta}_o$; the damping torque, $C\dot{\theta}_o$; and the spring restraint torque, $K\theta_o$. The ideal equation of motion may then be written as

$$J\ddot{\theta}_o + C\dot{\theta}_o + K\theta_o = H\omega_i \tag{A-1}$$

The solution of the reduced equation gives the response to a step angular input:

$$\theta_o = A \exp\left[\left(-\frac{C}{2J} \pm \sqrt{\frac{C^2 - 4JK}{4J^2}}\right)t\right] \tag{A-2}$$

For an integrating rate gyro,

$$C^2 \gg 4JK \tag{A-3}$$

Thus (A-2) may be written

$$\theta_o = A \exp\left(-\frac{t}{\tau}\right) \quad \text{approximately,} \tag{A-4}$$

where the time constant is

$$\tau = \frac{J}{C} \tag{A-5}$$

For the steady state, with the approximation (A-3), (A-1) becomes

$$C\dot{\theta}_o = H\omega_i$$

OR

$$\frac{\theta_o}{\theta_i} = \frac{H}{C} \equiv G_m \qquad \text{(A-6)}$$

the mechanical gyro transfer function. If the signal generator output voltage is proportional to the gimbal angle, so that

$$k_s = \frac{E_o}{\theta_o} \qquad \text{(A-7)}$$

then from (A-6)

$$\frac{E_o}{\theta_i} = \frac{H}{C} k_s \equiv G_s \qquad \text{(A-8)}$$

the signal generator gyro transfer function.

If the torque generator scale factor is defined as the ratio of the torque to the command torquer current,

$$k_T \equiv \frac{T_T}{I_T} \qquad \text{(A-9)}$$

and the gyro is operated in a loop closed through the torquer, then the torque applied by the torquer must equal the input torque.

$$T_T = H\omega_i$$

or

$$I_T k_T = H\omega_i \qquad \text{(A-10)}$$

The torquer transfer function is defined as

$$G_{TT} \equiv \frac{\omega_i}{I_T} = \frac{k_T}{H} \qquad \text{(A-11)}$$

Returning to equation (A-1) and solving in operator form,

$$\theta_o = \frac{A}{s^2 + \frac{C}{J}s + \frac{K}{J}} \qquad \text{(A-12)}$$

Since it is convenient in servomechanisms to express the coefficients of second order systems in terms of the damping ratio and the undamped natural frequency, (A-12) may be rewritten

$$\theta_o = \frac{A}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \text{(A-13)}$$

from which

$$\omega_n = \sqrt{\frac{K}{J}} \qquad \text{(A-14)}$$

and

$$\zeta = \frac{C}{\sqrt{4JK}} \qquad \text{(A-15)}$$

APPENDIX B

The Standard Deviation (σ)

The standard deviation is a much maligned term in inertial navigation, particularly with regard to drift rates.

The standard deviation is defined as the second moment about the mean. In this respect, the standard deviation is simply the moment of inertia about the mean of the area under the frequency distribution curve of the data. Algebraically, the standard deviation is defined by the equation:

$$\sigma = \left[\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n} \right]^{1/2} \quad \text{(B-1)}$$

where the x_i are the various data points, $\bar{x}$ is the arithmetic mean of the x_i, and n is the number of data points. Note that the algebraic form is similar to that for the rms deviation of a set of data from expected value or values.

The interpretation placed on the standard deviation must assume some form of the frequency distribution curve. A truly random distribution about a mean value will have a distribution curve known as normal, or Gaussian (Figure 4). The significance of the standard deviation for this curve, if the number of samples is large, is that approximately 68% of all values will fall within $\pm \sigma$ of the mean, 95% will fall within $\pm 2\sigma$ of the mean, and 99% will fall within $\pm 3\sigma$ of the mean. Figure 5 is a curve showing the percentage falling in the limits of $\pm y\sigma$ from the mean for a Gaussian distribution. If the frequency distribution is not Gaussian, then these interpretations cannot be applied. If no evidence indicates other than a truly random distribution about the mean, however, the Gaussian distribution may be assumed.

The above interpretation of the standard deviation is based on a large number of samples. It may be shown that the standard deviation for a sample of size n will deviate from the standard deviation of an infinite sample size for the same Gaussian distribution. This deviation is expressed by

$$\sigma_\infty = \sqrt{\frac{n}{n-1}}\ \sigma_n \quad \text{(B-2)}$$

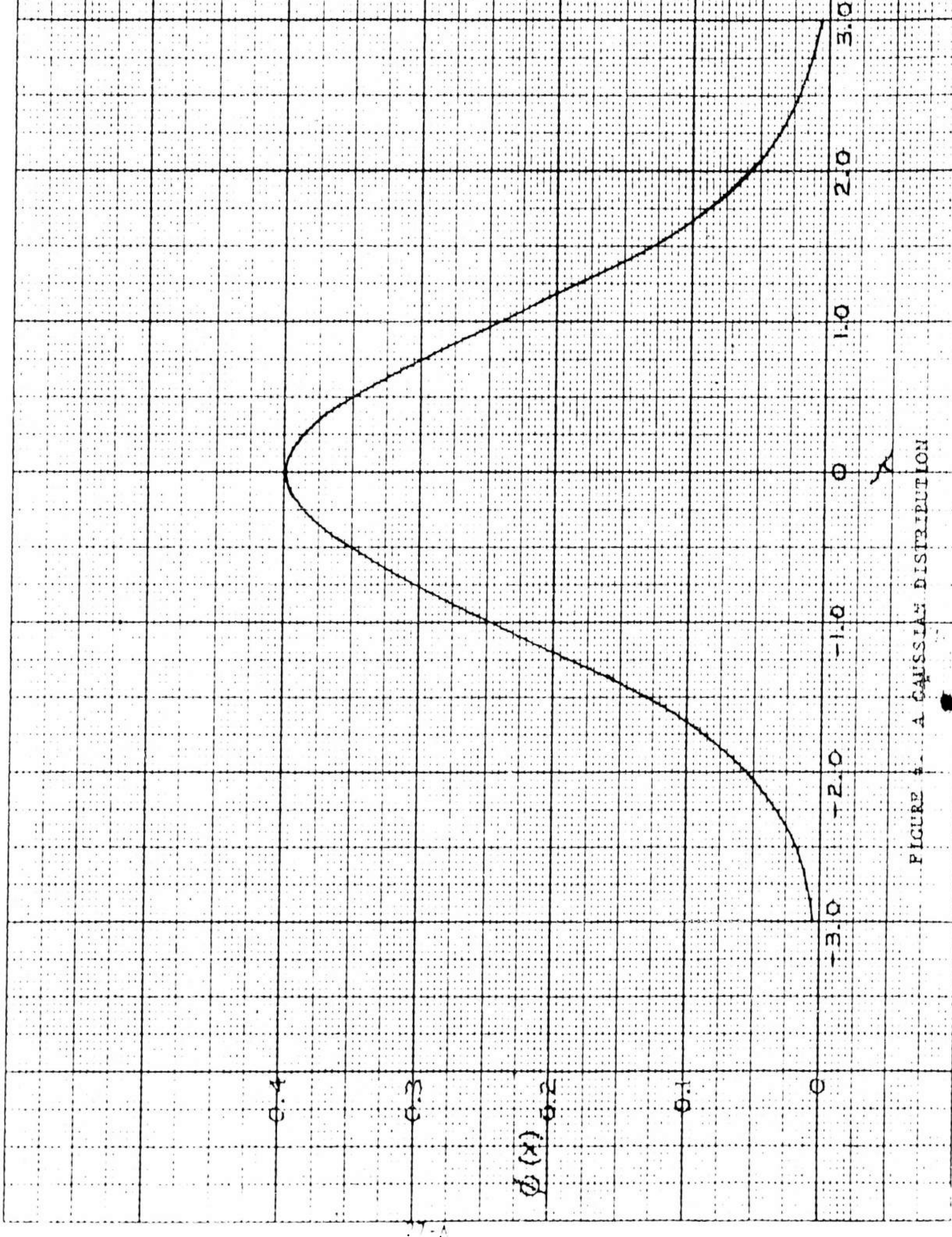

FIGURE [illegible]. A GAUSSIAN DISTRIBUTION

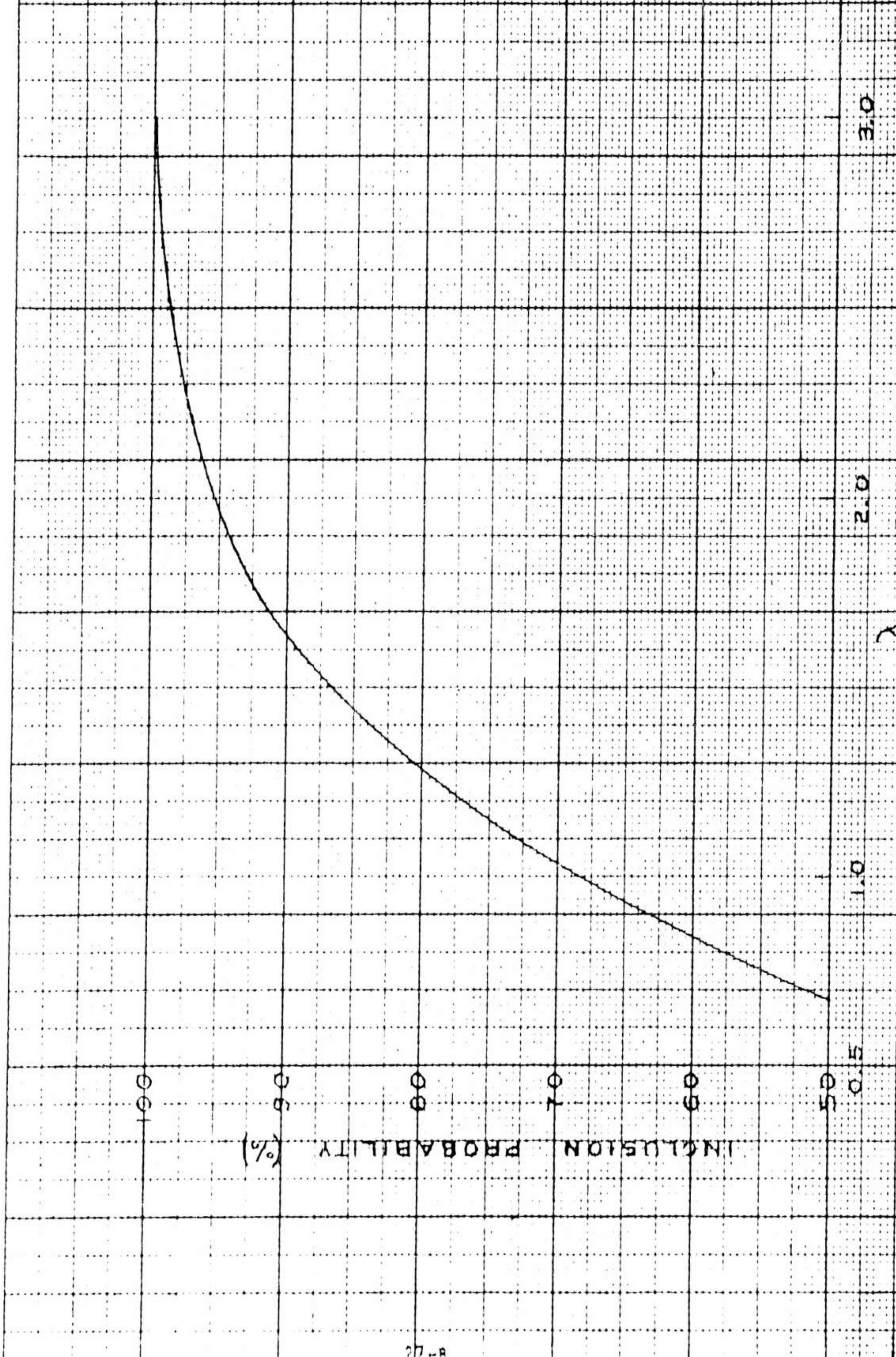

FIGURE 5. GRAPH OF INCLUSION PROBABILITY WITHIN A GIVEN NUMBER OF STANDARD DEVIATIONS

with the above interpretation of the standard deviation, it is desirable that equation (B-1) be adjusted so that is is more representative of a large sample. Thus, from (B-1) and (B-2),

$$\sigma_\infty = \sqrt{\frac{n}{n-1}}\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} \qquad \text{(B-3)}$$

The difference between the form (B-1) and (B-3) is small for large . If $r > 10$, the difference is less than 6%.

APPENDIX C

Non-Linearity And Random Effects

As in the torquer transfer function test, it may be desired to measure the non-linearity of a device where other random effects are also present, such as random drift. At any one point, then, it is impossible to isolate the contributions of torquer non-linearity from random drift effects, both which appear as non-linearities. The following derivation leads to a statistical method of separation.

Two basic assumptions are made: that the non-linearity of the torquer is randomly distributed with respect to the least squares line, which is one aim of the least squares method, and that the standard deviation of the random drift is representative of the technique and time intervals used in the torquer transfer function test.

If the total linearity error at the i^{th} data point is d_{ti} due to contributions from the non-linearity of the torquer d_{Li}, and from the instantaneous error in the average drift d_{Di}, then

$$d_{Ti} = d_{Li} + d_{Di} \qquad \text{(C-1)}$$

$$\therefore \quad d_{Ti}^2 = d_{Li}^2 + d_{Di}^2 + 2 d_{Li} d_{Di} \qquad \text{(C-2)}$$

Averaging over n data points

$$\frac{\sum^n (d_{Ti})^2}{n} = \frac{\sum^n (d_{Li})^2}{n} + \frac{\sum^n (d_{Di})^2}{n} + 2\,\frac{\sum^n (d_{Li} d_{Di})}{n} \qquad \text{(C-3)}$$

But

$$2\,\frac{\sum^n (d_{Li} d_{Di})}{n} = 2 \sum_{j=1}^{n} d_{Lj} \frac{\sum_{i=1}^{n} d_{Di}}{n}\, \delta_{ij} \qquad \text{(C-4)}$$

where δ_{ij} is the Kroenecker delta, having the value 1 when $i = j$, and the value 0 when $i \neq j$.

Now
$$\frac{\sum_{i=1}^{n} d_{Di}}{n} = 0 \tag{C-5}$$

since this is the average value of the drift, which is zero for the orientation of the gyro in the torquer transfer function test. Thus, (C-3) becomes

$$\frac{\sum_{i=1}^{n} (d_{Ti})^2}{n} = \frac{\sum_{i=1}^{n} (d_{Li})^2}{n} + \frac{\sum_{i=1}^{n} (d_{Di})^2}{n} \tag{C-6}$$

which by definition of the standard deviation, may be written

$$\sigma_T^{\,2} = \sigma_L^{\,2} + \sigma_D^{\,2} \tag{C-7}$$

Thus, if σ_T is calculated from the data, and σ_D is known from a drift test, then σ_L, the rms non-linearity of the torquer, may be calculated.

APPENDIX D

Least Squares Method of Determining The Best Linear Fit To A Set Of Data

Where a linear fit is desired to a set of data in two variables, the method of least squares provides a rather simple technique. The method assures that the sum of the squares of the deviations of the data from the straight line obtained is a minimum. It is not sufficient that the sum of the deviations be a minimum (zero), since any line passing through the point represented by the average values of the variables fulfills this condition. The least squares method assures that this second condition is fulfilled also.

The algebraic representations of these conditions are called the normal equations, which are

$$\sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i \tag{D-1}$$

and

$$\sum_{i=1}^{n} x_i y_i = a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 \tag{D-2}$$

where a and b are the intercept and slope of the desired line,

$$y_i = a + bx_i \tag{D-3}$$

Solving (C-1) and (C-2)

$$a = \bar{y} - b\bar{x} \tag{D-4}$$

where $\bar{x}$ and $\bar{y}$ are the average values of x and y, and

$$b = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2} \tag{D-5}$$

A typical calculation is shown in Table I , with the data and least squares line plotted in figure 6 . Also shown in the figure is a line which was fitted to the data by the "eyeball" method.

The best check against the calculations performed in determining the coefficients a and b is to plot the raw data and plot equation (D-3). If a gross error in the calculations exists, it will be immediately obvious from the lack of fit to the raw data.

TABLE I

DATA AND LEAST SQUARES CALCULATIONS FOR FIGURE 6

X	Y
0.5	3.5
1.0	5.0
1.0	6.5
2.5	8.0
2.5	11.0
3.5	9.0
4.0	12.5
5.0	15.0
5.5	12.5
8.0	17.0

$n = 10$
$\sum x = 33.5$
$\bar{x} = 3.35$
$n\bar{x}^2 = 112.225$
$\sum y = 99.5$
$\bar{y} = 9.95$
$\sum xy = 420.0$
$n\bar{x}\bar{y} = 333.325$
$\sum x^2 = 162.25$

$b = 1.7326$
$a = 4.1458$

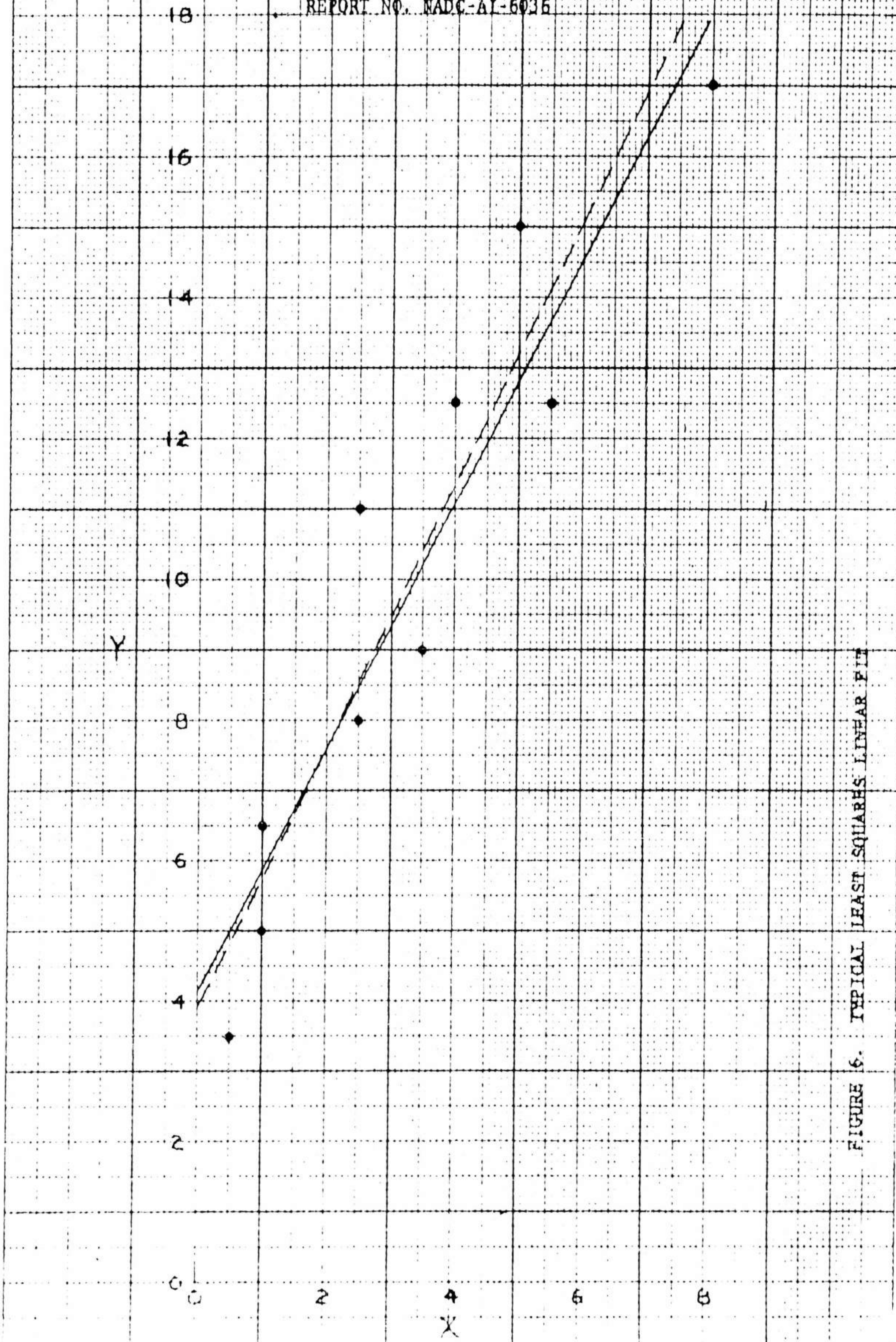

FIGURE 6. TYPICAL LEAST SQUARES LINEAR FIT

APPENDIX E

Polynomial Regression

It may be desired to fit a curve of higher degree than a straight line to a set of data. The least squares method used in Appendix D can also be applied to other algebraic forms. In such curves as exponential and logarithmic functions, the linear least squares method may be used with the proper substitution of variables.

For the polynomial form

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \qquad (E-1)$$

the normal equations are similar to those given in (C-1).

$$
\begin{aligned}
a_0 n + a_1 \sum x + a_2 \sum x^2 + \cdots + a_n \sum x^n &= \sum y \\
a_0 \sum x + a_1 \sum x^2 + a_2 \sum x^3 + \cdots + a_n \sum x^{n+1} &= \sum xy \\
\text{[illegible]} & \qquad (E-2) \\
a_0 \sum x^n + a_1 \sum x^{n+1} + a_2 \sum x^{n+2} + \cdots + a_n \sum x^{2n} &= \sum x^n y
\end{aligned}
$$

For the quadratic form used in the calculation of the non-Newtonian behaviour of the damping fluid, the solution of (E-2) for the parameter values gives

[illegible]

$$\text{where } D = n\left(\Sigma x^2\Sigma x^4 - \Sigma x^3\Sigma x^3\right) + \Sigma x\left(\Sigma x^3\Sigma x^2 - \Sigma x\Sigma x^4\right) + \Sigma x^2\left(\Sigma x\Sigma x^3 - \Sigma x^2\Sigma x^2\right) \tag{E-4}$$

$$N_0 = \Sigma y\left(\Sigma x^2\Sigma x^4 - \Sigma x^3\Sigma x^3\right) + \Sigma x\left(\Sigma x^3\Sigma x^2 y - \Sigma xy\Sigma x^4\right) + \Sigma x^2\left(\Sigma xy\Sigma x^3 - \Sigma x^2\Sigma x^2 y\right) \tag{E-5}$$

$$N_1 = n\left(\Sigma xy\Sigma x^4 - \Sigma x^3\Sigma x^2 y\right) + \Sigma y\left(\Sigma x^3\Sigma x^2 - \Sigma x\Sigma x^4\right) + \Sigma x^2\left(\Sigma x\Sigma x^2 y - \Sigma xy\Sigma x^2\right) \tag{E-6}$$

$$N_2 = n\left(\Sigma x^2\Sigma x^2 y - \Sigma x^3\Sigma xy\right) + \Sigma x\left(\Sigma xy\Sigma x^2 - \Sigma x\Sigma x^2 y\right) + \Sigma y\left(\Sigma x\Sigma x^3 - \Sigma x^2\Sigma x^2\right) \tag{E-7}$$

This formidable set of equations actually contains only the following elements:

n	Σx^4
Σx	Σy
Σx^2	Σxy
Σx^3	$\Sigma x^2 y$

Of these, only the following have not been calculated in the solution for the damping coefficient by calculating the least squares line:

$$\Sigma x^3$$
$$\Sigma x^4$$
$$\Sigma x^2 y$$

Thus, the calculations are not as tedious as they might at first seem.

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Report on Test Methods for the Single Degree of Freedom Integrating Rate Gyro
Phase I: Gyro Characteristics, by R. Vaughn,
37 p, incl table & figs., Jun 1960

General methods and special techniques for testing single degree of freedom gyros have been studied. This report deals with those tests which determine gyro design characteristics, such as damping coefficient, signal generator scale factor, torque generator scale factor, input axis alignment, and others. The tests are designed to be performed on a completed gyro without breaking the seal.

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